SPECTRAL CONTENT OF NRZ TEST PATTERNS

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Introduction

Nonreturn-to-zero (NRZ) signaling is widely used for data transmission in digital communication systems. Many NRZ test patterns have been created for the purpose of system test and verification. These patterns are usually designed to either simulate actual data or to stress certain aspects of the system. In order to understand the effects of the various test patterns on a particular system, it is important to understand the frequency characteristics of both the test pattern and the system under test.

The purpose of this article is to show straightforward relationships between the time-domain characteristics of NRZ test patterns (such as data rate and pattern length) and their frequency-domain spectral components. Topics include an overview of NRZ test patterns, computation of the power spectrum, lab measurements of the power spectrum, and application of these concepts to system understanding.

Overview of NRZ Test Patterns

In NRZ signaling, each binary bit is assigned a unique time slot of duration T_b, called the bit period. The signal is either high (representing a one) or low (representing a zero) during the entire bit period. NRZ waveforms are defined and measured as functions of time, i.e., they are inherently time-domain signals.

For a random NRZ data stream, each bit in the sequence has an equal probability (50%) of being a one or a zero regardless of the state of the preceding bit(s). Therefore, there exists the possibility of having large sequences of consecutive identical digits (CIDs). Designing high-speed systems that can work with random data can be difficult due to the very low frequency content produced by the long sequences of CIDs in the data signal.

Data encoding or scrambling is often used to format the random data into a more manageable form. One of the most widely used encoding methods in high-speed systems is known as 8b10b, which is used in Ethernet, Fibre Channel and high-speed video applications. 8b10b encoding takes 8 bits of data and replaces it with a 10-bit symbol. The extra bits are added to balance the pattern (make number of ones equal the number of zeros for a given bit interval) and limit the maximum number of CIDs. The encoding algorithm is also used to improve bit error ratio (BER) by mapping the 8 bits to specific symbols in the 10-bit signal space that can be easily distinguished from other 10-bit symbols. Other methods such as scrambling or 64b66b encoding are common to SONET and SDH telecommunication systems. Scrambling and 64b66b encoding also work to balance the pattern and improve the BER but much larger runs of CIDs are possible with these methods.

For a given application, there may be several types of test patterns that stress various performance aspects or components of the system. For example, a K28.5± pattern (11000001010011111010) is often used to test the deterministic jitter performance of systems that use 8b10b encoding while a pseudo-random bit stream (PRBS) is used as a general purpose test pattern in encoded, random and scrambled NRZ applications.

The PRBS is typically denoted as a 2^X-1 PRBS. The power (X) indicates the shift register length that is used to create the pattern. Each 2^X-1 PRBS contains every possible combination of X number of bits (except one). A short PRBS such as the 2^7-1 PRBS (127 Bits) is often used in Ethernet, Fibre Channel and high-speed video applications as it provides a good approximation to an 8b10b encoded NRZ data stream. A 2^{23}-1 (≈ 8.4 Million Bits) PRBS is commonly used in SONET and SDH telecommunication systems that require a test pattern with lower frequency content and that provides a better representation of scrambled or random NRZ data.

Computing the Power Spectrum of an NRZ Test Pattern

Each nonreturn-to-zero (NRZ) test pattern has an associated power spectral density (PSD) that serves as an indication of the frequency distribution of the power in the pattern. The two primary methods of computing PSD are: (a) Square the magnitude of the Fourier transform of the pattern, or (b) Compute the Fourier transform of the autocorrelation function of the pattern[1]. The first method is generally simpler for signals that can be mathematically written in a finite, closed form (e.g., $s[t] = A\cos[2\pi f_0 t]$), and the second method is used for more complicated signals, such as long sequences of NRZ data (like test patterns) or random bit streams. In order to apply these methods, it is useful to review some of the basics of Fourier analysis[2].

1. The delta function, $A\delta(t)$, can be thought of as an infinitely narrow rectangular pulse with area A. It has a non-zero value only when the argument of the function is equal to zero and is represented graphically by a vertical arrow.
2. The comb function, $A\,\Sigma_n\delta(t-nT)$ is made up of an infinite number of equal area delta functions spaced at uniform intervals, T.
3. The Fourier transform of a comb function is also a comb function, where the interval is inverted (e.g., n/T) and the areas of the delta functions are modified by the inverted interval (e.g., A/T).
4. Convolution in the time domain (represented symbolically by *) is equivalent to multiplication in the frequency domain, and visa versa.
5. Convolution of a signal with a delta function results in a copy of the signal that is shifted to the location of the delta function.
6. Multiplication of a signal with a delta function ("sampling") results in a delta function with its area modified by the magnitude of the signal at the location of the delta function.

As an example of the application of the above rules, we compute the PSD of an NRZ test pattern (Figure 1). The test pattern can be represented by a sequence of high and low levels (representing ones and zeros) with a defined bit period, T_b, and total pattern length, $L = nT_b$. Convolution of the finite length test pattern with a comb function that has a spacing interval equal to the pattern length results in an infinite repetition of the pattern (Figure 1.a). Next we separately compute the autocorrelation functions for each component of the test pattern (Figure 1.b) and note that the autocorrelation of the test pattern approximates a triangle (the accuracy of this approximation improves as the length and "randomness" of the pattern increase). Finally, we use the Fourier transform of the autocorrelation functions to compute the power spectrum (Figure 1.c).

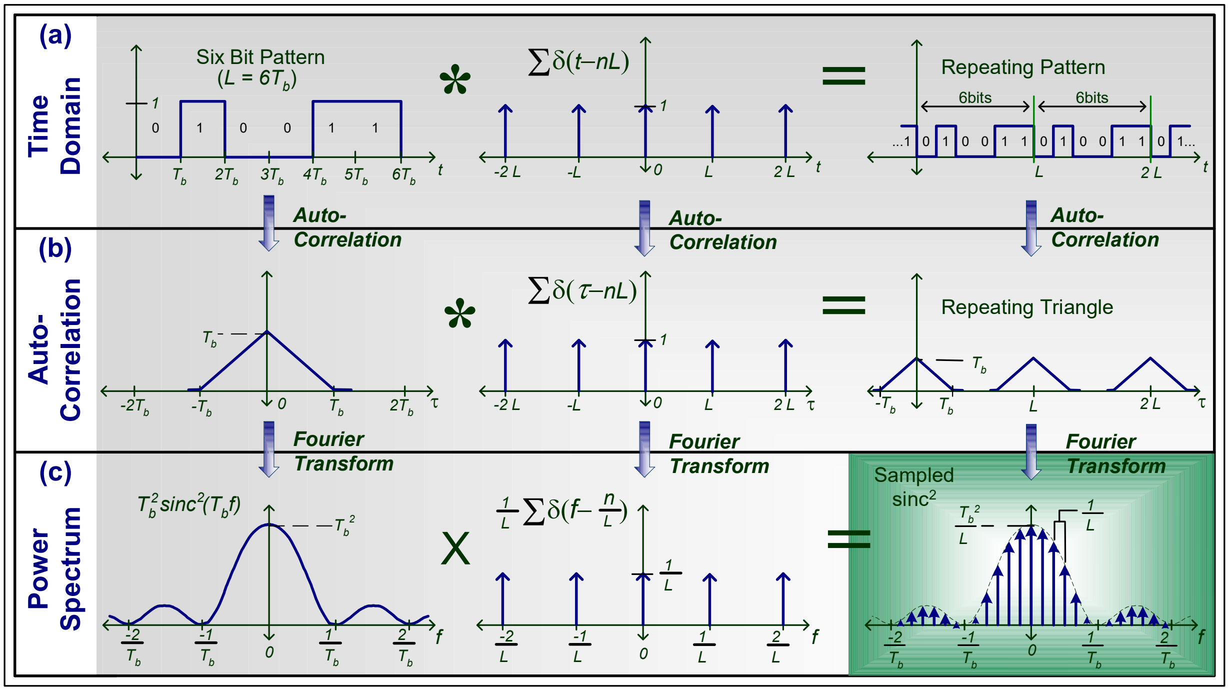

Figure 1. The Time Domain (a), Auto-Correlation (b) and Power Spectrum (c) of NRZ.

The power spectrum resulting from the above example shows an infinite sequence of discrete spectral lines (delta functions) scaled by a "$\text{sinc}^2(f)$" envelope, where sinc(f) is defined as $\sin(\pi f)/(\pi f)$. Important observations that apply to test patterns in general include the following: (a) The nulls in the $\text{sinc}^2(f)$ envelope occur at integer multiples of the data rate, (b) Spectral lines are evenly spaced at an interval that is the inverse of the pattern length, and (c) The magnitude of the $\text{sinc}^2(f)$ envelope decreases (i.e., "flattens out") as the data rate and/or pattern length are increased. In the limit, as the pattern length approaches infinity, the spacing between the spectral lines becomes infinitesimally small and the spectrum shape approaches a continuous $\text{sinc}^2(f)$ function.

As an example, if the six-bit pattern shown in figure 1.a is transmitted at a data rate of 1.25Gbps, the spectral line spacing, amplitude, and spectral nulls can be calculated as shown in figure 2. Note that $\text{sinc}^2(f)$ envelope shown in figure 2 is an approximation of the six-bit pattern. The accuracy of this approximation improves as the pattern length / randomness increases.

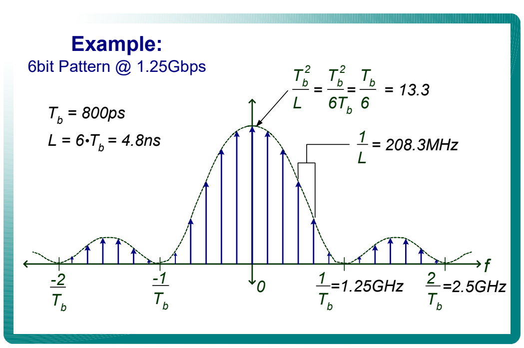

Figure 2: Approximate Power Spectrum of a 6-bit NRZ pattern showing spectral line spacing and the $\text{sinc}^2(f)$ envelope.

Spectrum Analyzer Measurements

The equations and principles described above can also be demonstrated by lab measurement using a high-speed pattern generator to create the test patterns and a spectrum analyzer to measure the power spectral density of the signal. Starting with a simple example, the measured spectrum of a 4-bit pattern (1110) transmitted at 1.25Gbps can be seen in figure 3. The spectral nulls are measured at 1.25GHz ($1/T_b$) and 2.5GHz ($2/T_b$) and the line spacing is 312.5GHz (1/L). The power spectrum envelope is also seen to be approximately $sinc^2(f)$. The slight deviations in magnitude are the result of the short pattern used in this example.

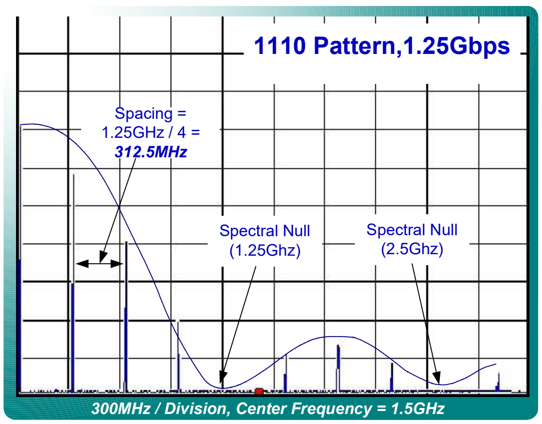

Figure3: The measured Power Spectrum of a 4-bit pattern shows slight deviations of the spectral line magnitude from the $sinc^2(f)$ envelope. As the pattern increases in length the deviation will reduce.

Increasing the pattern length to 20 bits (K28.5± Test Pattern) and keeping the transmission rate at 1.25Gbps (Figure 4), the spectral nulls are measured to be in the same locations (1.25GHz and 2.5GHz) while the spectral line spacing is reduced to 125MHz due to the longer pattern length. The $sinc^2(f)$ envelope of the spectral lines is also seen to be a more accurate representation. The envelope of the spectral lines is also seen to be more closely matched to the $sinc^2(f)$ function than the 4-bit pattern example.

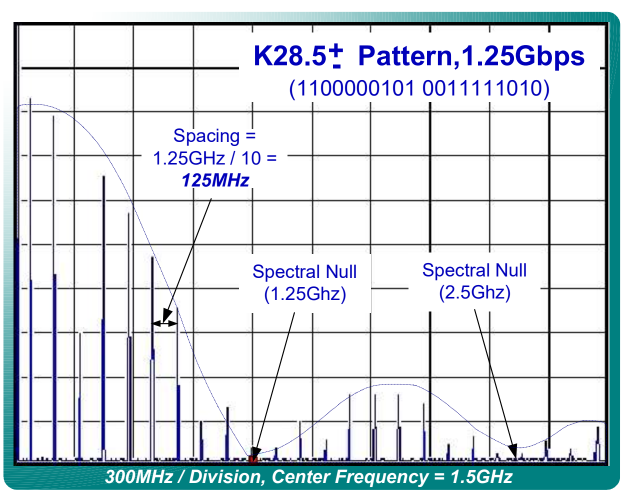

Figure4: The measured power spectrum of a K28.5± test pattern shows the improved approximation of the $sinc^2(f)$ envelope and the reduced spectral line spacing given the longer pattern.

The K28.5± test pattern presents an interesting aspect to this topic as it is a 20-bit pattern but the spectral line spacing is measured to be 125MHz when transmitted at 1.25Gbps, which would correspond to a 10-bit test pattern. This discrepancy is due to the fact that the K28.5± pattern is composed of a k28.5+ sequence (1100000101) and its inverse, the K28.5- sequence (0011111010). In the frequency domain the K28.5- sequence contains the same spectral information as the K28.5+ sequence. The pattern therefore repeats spectrally every 10 bits and is the reason for the 125MHz spacing in the above example.

The $sinc^2(f)$ envelope becomes very apparent as the pattern length is increased further. Figure 5 illustrates this point using a 2^7-1 PRBS pattern (127 bits) transmitted at 2.5Gbps. With this longer pattern length, the delta spacing is reduced to approximately 19.7MHz and the spectral nulls are at 2.5GHz and 5GHz corresponding to the higher data rate. Given the small spectral line spacing in respect to the data rate, the $sinc^2(f)$ envelope and spectral nulls are clearly seen in the power spectrum (Figure 5).

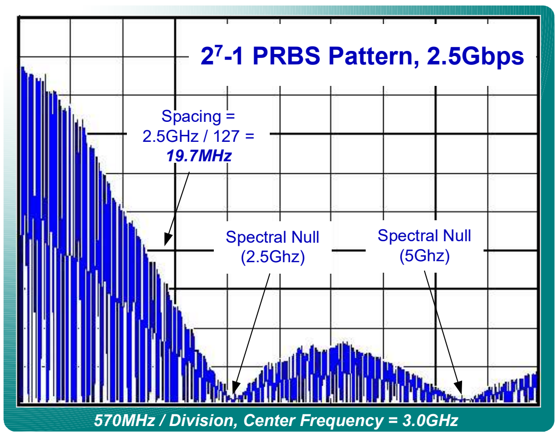

Figure5: The power spectrum of the 2^7-1 PRBS (127bits) clearly shows the spectral nulls and $sinc^2(f)$ envelope.

Figure 6 illustrates the difference in the spectral line magnitude and spacing for a 2^7-1 PRBS pattern at 1.25Gbps and 2.5Gbps. As seen in this figure, when measured at the same frequency, the magnitude of the spectral lines and the spacing is larger at 2.5Gbps data transmission rates than at 1.25Gbps.

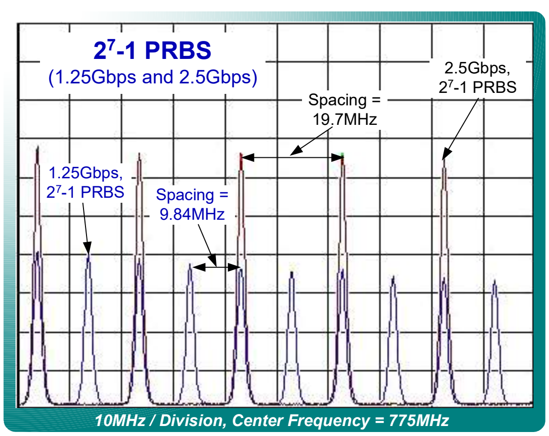

Figure 6: The measured power spectrum of a 2^7-1 PRBS Pattern transmitted at 1.25 & 2.5Gbps (Viewed at 725MHz to 825MHz) shows the spectral line magnitude and spacing difference as the data rate is changed.

Example Applications

Knowledge of the power spectrum of NRZ test patterns can lead to significant improvements in the design of digital communication systems. This is illustrated through examples of three different applications: receiver bandwidth, adaptive equalizers, and electromagnetic interference (EMI).

Receiver Bandwidth: The design process for a receiver inevitably includes questions about the necessary bandwidth. If the bandwidth is too low, the high-frequency components of the received signal will be attenuated, and the signal will be distorted. If the bandwidth is too high, excess noise will be admitted to the receiver causing a reduction in signal-to-noise ratio (SNR) along with the increased complexity and cost necessary to achieve the higher bandwidth[3]. Armed with the knowledge of the spectral content of the signals that will be received, the bandwidth decision can be made in a manner that includes the critical spectral components, but no more.

Adaptive Equalizer: Adaptive equalizers are designed to reverse distortion effects caused by non-ideal transmission media. The MAX3800 adaptive cable equalizer, for example, is designed to reverse the distortion caused by skin effect losses in copper cables at data rates as high as 3.2Gbps[4]. It accomplishes this by comparing the power in the input signal at two discrete frequencies (f_1 = 200MHz and f_2 = 600MHz). Based on the $\text{sinc}^2(T_b f)$ envelope of the power spectrum with the first null at 3.2GHz, the ratio of the power at these two frequencies should be $\text{sinc}^2(T_b f_1)$ / $\text{sinc}^2(T_b f_2)$ = 0.987/0.890 = 1.11. If the measured ratio is different than expected, the equalizer changes the amount of skin effect compensation in order to restore the correct ratio. This works well for high data rates and long data patterns, but, using our knowledge of spectral content of NRZ test patterns, we can predict that some patterns may cause problems.

For example, if the data rate is reduced to 622Mbps, the $\text{sinc}^2(f)$ envelope with first null at 622MHz will result in a 200MHz to 600MHz power detector ratio of 0.703/0.00134 = 525 instead of the expected 1.11. As the equalizer tries to restore the expected 1.11 power ratio, the output may be distorted. As another example, consider a short test pattern with a pattern length of 10 bits. For shorter patterns the spectral lines will be spaced at larger intervals. In the specific case of the 10bit pattern at a data rate of 3.2Gbps, the spectral lines will be separated by 320MHz, with the first few at 0, 320, and 640MHz. For this type of pattern and data rate, there may be little or no power to detect at 200MHz or 600MHz, which can cause signal distortion due to the equalizer not being able to adapt correctly.

Electromagnetic Interference (EMI): The effects of EMI in a system can be reduced or eliminated by altering the magnitude and/or frequencies of the power spectrum. This can be done by changing the data rate or the pattern length.

As the data rate increases, the nulls of the spectrum are spread farther apart and the magnitude of each spectral line is reduced by pushing some of the power to higher frequencies. Spreading the power over a larger frequency range will leave less at the frequencies of interest. One way to achieve this effect is by adding extra bits to the original data stream to effectively increase the data rate.

Pattern length also plays a role in EMI since spectral line magnitude and spacing vary as the pattern length is changed. A longer pattern will reduce the magnitude and spacing, while a shorter pattern will increase the magnitude and spacing. To reduce EMI at a specific frequency, the pattern length can be changed to shift the spectral line away from a particularly sensitive frequency range, or a longer pattern can be used to reduce the magnitude of the EMI.

Conclusion

A clear understanding of the frequency domain spectral content of NRZ data is critical to success in high-speed digital communication system design. The principles presented in this article establish basic relationships between NRZ data time-domain characteristics (pattern length, data rate, etc.) and their corresponding frequency domain characteristics (spectral magnitude, envelope and line spacing). These principles can be applied to a variety of circuit design issues including filtering, signal equalization, EMI and more.

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